

# **No Information Sharing in Oligopoly: The Case of Price Competition with Cost Uncertainty**

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## **Abstract**

We show that concealing cost information is a dominant strategy in heterogeneous Bertrand oligopolies. This result enables us to endogenize the number of firms in a market in terms of market size, entry costs, and unit cost uncertainty.

JEL classification: L13, D43, D82, C72, C73

## **Zusammenfassung**

### **Kein Informationsaustausch im Oligopol: Der Fall von Preiswettbewerb bei Kostenunsicherheit**

In diesem Beitrag zeigen wir, daß für ein heterogenes Oligopol die dominante Strategie der Unternehmen bei Preiswettbewerb lautet, keine Informationen über die Höhe der Produktionsstückkosten auszutauschen. Auf der Basis dieses Ergebnisses läßt sich die Anzahl der Unternehmen im Markt in Abhängigkeit vom Marktvolumen, von den Kosten des Marktzutritts und von der Unsicherheit über die Höhe der Produktionsstückkosten endogenisieren.

JEL-Klassifikation: L13, D43, D82, C72, C73

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# 1 Introduction

In most markets in which firms interact strategically, firms are better informed about their own cost and demand parameters than about those of their rivals. It is therefore an important issue in Industrial Organization theory to study the incentives of firms to exchange private information (see, e.g., *Vives* 1999, ch. 8). The literature on information sharing in oligopoly is vast. However, most papers study Cournot competition in homogeneous markets. Even if in reality it is obviously the most important kind of tactical competition, only few authors deal with price competition in heterogeneous markets. While *Vives* (1984) and *Sakai* (1986) concentrate on the exchange of demand information, *Gal-Or* (1986) and *Sakai* (1991) study the expected gains of exchanging cost information. These Bertrand duopoly models yield the well known result that firms reveal their private information in Bayesian Nash equilibrium under demand uncertainty, while under cost uncertainty they do not. In an influential paper, *Raith* (1996) develops a unified approach in which he derives the conclusions of the models cited above as special cases.

While it is generally accepted in the literature that the duopoly results with demand uncertainty generalize to oligopolistic market structures, *Raith* (1996, p. 279) argues that with cost uncertainty “... *results obtained for duopolies do not extend to larger markets.*” This would indicate that the concealing strategy is not very robust. Instead, for a large number of firms in the market, unilateral revelation of private cost information should not be a dominant strategy. The present paper deals with the case of price competition and cost uncertainty of *Raith’s* (1996) generalized model, correcting a far-reaching error and drawing a new conclusion for Bertrand oligopolies with cost uncertainty. We derive that concealing cost information is a dominant strategy in price competition with substitutive goods. This unambiguous conclusion enables us to endogenize the number of firms in heterogeneous markets in terms of market size, entry costs as well as in terms of cost uncertainty. We show that either expected equilibrium profits or the equilibrium number of firms in the market increase with cost uncertainty.

## 2 The Model

We consider a market consisting of  $n \geq 2$  firms, each producing a differentiated good. According to most models in the Industrial Organization literature, we assume a quadratic utility function

$$U(q_0, \mathbf{q}) = q_0 + \alpha \sum_{i=1}^n q_i - \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \beta_{ij} q_i q_j \quad (1)$$

of a representative consumer. Each consumer demands  $q_0$  units of the numéraire good and  $q_i$  units of the differentiated goods  $i = 1, \dots, n$ , represented by the vector  $\mathbf{q} = (q_1, \dots, q_n)'$ . We impose the usual parameter restrictions  $\beta_{ij} = \beta \forall i = j$ ,  $\beta_{ij} = \gamma \forall i \neq j$ ,  $\alpha > 0$  and  $0 < \gamma < \beta$ . Consumers maximize utility subject to the budget constraint

$$q_0 + \sum_{i=1}^n p_i q_i \leq I, \quad (2)$$

where  $I$  denotes income,  $\mathbf{p} = (p_1, \dots, p_i, \dots, p_n)'$  the price vector of goods  $i = 1, \dots, n$ , and the price of the numéraire good is normalized to one. The first-order conditions determining the optimal consumption levels of all goods lead to the linear demand functions

$$D_i(\mathbf{p}) = a - bp_i + d \sum_{j \neq i} p_j, \quad i, j = 1, \dots, n \quad (3)$$

where  $a := \frac{\alpha}{\beta + (n-1)\gamma}$ ,  $b := \frac{\beta + (n-2)\gamma}{(\beta - \gamma)[\beta + (n-1)\gamma]}$  and  $d := \frac{\gamma}{(\beta - \gamma)[\beta + (n-1)\gamma]}$ , which implies  $0 < (n+1)d < b$ .

We assume the same type of cost uncertainty as in *Gal-Or* (1986), i.e., firms know the distribution function of their unit cost, but are only imperfectly informed about the realizations. The deviations  $\tau_i$  from the expected value  $c$  are independently and identically normally distributed with means zero and variances  $t \geq 0$ , i.e.  $\tau_i \sim N(0, t)$ .<sup>1</sup>

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<sup>1</sup> There may be parameter constellations where the nonnegativity constraint of unit cost is not fulfilled if the random variables are assumed to be normally distributed. However, this distribution function which is usually applied in the information exchange literature can be interpreted

If firms knew the realization of their respective deviation parameters  $\tau_i$ , but were uncertain about the rivals' cost parameters, the underlying information structure would be a standard one of asymmetric information. However, the assessment of the advantage or disadvantage of information exchange becomes more complicated if firms have to decide about their information revelation behavior at a point in time when information about their own unit cost is also uncertain. As in the *Gal-Or* (1986) model, we simultaneously analyze both stochastic uncertainty of firms about their own unit cost as well as asymmetric information between the competitors.

Firm  $i$ 's ex ante observed signal for the deviation parameter  $\tau_i$  is  $\varphi_i = \tau_i + \psi_i$ , where the signal errors  $\psi_i$  are also assumed to be independently and identically normally distributed with means zero and variances  $u \geq 0$ , i.e.  $\psi_i \sim N(0, u)$ . Thus, firms can make no inferences about the unit cost of their rivals based on their private cost information. However, each firm is able to signal its perception of unit cost to the rivals. In a very general way we may account for the precision of a strategic information revelation by specifying the signal as  $\hat{\varphi}_i = \varphi_i + \xi_i$ . The strategic revelation deviations  $\xi_i$  are also assumed to be independently and identically normally distributed random variables with means zero and variances  $r_i \geq 0$ , i.e.  $\xi_i \sim N(0, r_i)$ . If the signal is sent with zero variance ( $r_i = 0$ ), firm  $i$  perfectly reveals its cost information. In the case of an infinitely high variance ( $r_i \rightarrow \infty$ ), it conceals its private information.<sup>2</sup> All firms have to make their pricing decisions using all available cost information, represented by their respective information sets  $\mathbf{z}_i = (\varphi_i, \hat{\boldsymbol{\varphi}})'$  with the vector  $\hat{\boldsymbol{\varphi}}' = (\hat{\varphi}_1, \dots, \hat{\varphi}_n)$  of revealed information by all firms.

The ex ante expected profit function of firm  $i$  is

$$E^i [\pi^i(\mathbf{p}) | \mathbf{z}_i] = E^i \left\{ \left\{ [p_i - (c + \tau_i)] \left[ a - bp_i + d \sum_{j \neq i} p_j(\mathbf{z}_j) \right] \right\} \middle| \mathbf{z}_i \right\} \quad (4)$$

where  $E$  is the expected value operator. The necessary first-order conditions<sup>3</sup> lead

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as an approximation of any specific distribution function.

<sup>2</sup> In this way, strategic lying in the revelation process as modeled by *Ziv* (1993) is excluded. Firms only have the option to reveal their cost information with an arbitrarily large noise. This means that concealing occurs by announcing and sending worthless signals.

<sup>3</sup> The sufficient conditions for a profit maximum are globally met. In order to simplify the analysis, we generally assume parameter values that guarantee Bayesian Nash equilibria with positive quantities for all firms.

to the reaction functions:

$$p_i(\mathbf{z}_i) = \frac{a + b [c + E^i(\tau_i | \mathbf{z}_i)] + d \sum_{j \neq i} E^i [p_j(\mathbf{z}_j) | \mathbf{z}_i]}{2b} \quad (5)$$

Since the resulting equilibrium strategies are affine in the information sets  $\mathbf{z}_i$ , the proposed solution equations take the form:

$$p_i(\mathbf{z}_i) = \eta_{0i} + \eta_{1i} \varphi_i + \boldsymbol{\eta}_{2i}' \hat{\boldsymbol{\varphi}} \quad (6)$$

In order to solve for prices in Bayesian Nash equilibrium, we have to determine the coefficients  $\eta_{0i}, \eta_{1i} \in \mathbb{R}$  and  $\boldsymbol{\eta}_{2i} \in \mathbb{R}^n$ . For the expected price decisions of the competitors we obtain:

$$E^i [p_j(\mathbf{z}_j) | \mathbf{z}_i] = \eta_{0j} + \eta_{1j} E^i(\varphi_j | \mathbf{z}_i) + \boldsymbol{\eta}_{2j}' \hat{\boldsymbol{\varphi}}, \quad i, j = 1, \dots, n, \quad i \neq j \quad (7)$$

Due to the assumptions of the normal distributions, the conditional mean  $E^i(\varphi_j | \mathbf{z}_i)$  solves as  $E^i(\varphi_j | \mathbf{z}_i) = \frac{t+u}{t+u+r_j} \mathbf{e}_j' \hat{\boldsymbol{\varphi}}$ , where  $\mathbf{e}_j$  is the  $j$ -th unit vector. In an analogous way, the expected deviations of unit cost from their mean are  $E^i(\tau_i | \mathbf{z}_i) = \frac{t}{t+u} \varphi_i$ . By inserting these conditional means together with equation (7) into the reaction functions (5) and equating the resulting expression with the proposed solution equations (6), the coefficients can be identified as

$$\eta_{0i} = \frac{a + bc}{2b - (n-1)d} \quad (8)$$

$$\eta_{1i} = \frac{t}{2(t+u)} \quad (9)$$

$$\boldsymbol{\eta}_{2i} = t \left\{ \frac{bd}{(2b+d)[2b-(n-1)d]} \bar{\mathbf{m}} - \frac{d}{2(2b+d)(t+u+r_i)} \mathbf{e}_i \right\} \quad (10)$$

with vector  $\bar{\mathbf{m}} := \left( \frac{1}{t+r_1}, \dots, \frac{1}{t+r_n} \right)'$ . Consequently,  $\boldsymbol{\eta}_{2i}$  contains the elements:

$$\eta_{2ii} = \frac{(n-1)d^2t}{2(2b+d)[2b-(n-1)d](t+u+r_i)} \quad (11)$$

$$\eta_{2ij} = \frac{bdt}{2(2b+d)[2b-(n-1)d](t+u+r_j)} \quad \forall i \neq j \quad (12)$$

Inserting these expressions into the solution equations (6), we obtain the optimal

price strategies which yield the ex ante expected equilibrium profits:<sup>4</sup>

$$\begin{aligned}
E^i(\pi^i | \mathbf{z}_i) &= bE^i \{ [p_i - (c + \tau_i)]^2 \} \\
&= b \left\{ \left\{ \frac{a - [b - (n-1)d]c}{2b - (n-1)d} \right\}^2 + t^2 \left\{ \frac{t + 4u}{4t(t+u)} \right. \right. \\
&\quad \left. \left. + \frac{b^2 d^2}{(2b+d)^2 [2b - (n-1)d]^2} \sum_{j \neq i} \frac{1}{t+u+r_j} \right. \right. \\
&\quad \left. \left. - \frac{(n-1)d^2 [4b(2b+d) - (n-1)d(4b+3d)]}{4(2b+d)^2 [2b - (n-1)d]^2 (t+u+r_i)} \right\} \right\} \quad (13)
\end{aligned}$$

Differentiating these functions with respect to the revelation variances  $r_i$  yields:

$$\frac{dE^i(\pi^i | \mathbf{z}_i)}{dr_i} = \frac{(n-1)bd^2 [4b(2b+d) - (n-1)d(4b+3d)]}{4(2b+d)^2 [2b - (n-1)d]^2 (t+u+r_i)^2} > 0 \quad (14)$$

Since, as already noted,  $0 < (n-1)d < b$ , it can be shown that this derivative is positive. Therefore, in contrast to the corresponding result derived by *Raith* (1996, p. 279), firms will generally choose an infinite variance ( $r_i \rightarrow \infty$ ) in information transmission which is equivalent to concealing its private cost information. Using this concealing strategy, firms weaken the price competition and, hence, increase their expected profits.

Thus, *Gal-Or's* (1986) result for a duopolistic market indeed generalizes to oligopolistic market structures. Even for a very large number of firms, unilateral revelation of private cost information never constitutes a Bayesian equilibrium strategy.<sup>5</sup>

Consequently, using the solution equations (6) with the coefficients defined in equations (8) to (10), we obtain the equilibrium prices

$$p_i = \frac{a + bc}{2b - (n-1)d} + \frac{t}{2(t+u)} \varphi_i \quad (15)$$

and from (13) the expected equilibrium profits:

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<sup>4</sup> See appendix.

<sup>5</sup> This point was recently and independently made in a note by *Jin* (2000).

$$E(\pi^*) = b \left\{ \left\{ \frac{a - [b - (n-1)d]c}{2b - (n-1)d} \right\}^2 + \frac{t(t+4u)}{4(t+u)} \right\} \quad (16)$$

The presented model explains the empirical observation that firms generally refuse to reveal private cost information. As our analysis has shown, concealing successful efforts in obtaining process innovations should be an important firm strategy in order to temporarily maintain a technologically leading market position, especially if patent protection is not perfect.

### 3 Cost Uncertainty and Market Structure

The comparative statics, as analyzed by *Raith* (1996), are not valid for the assumed utility function, since the parameters he treats as exogenous are in fact endogenously determined by the number of firms. To show the effects of market structure on the firms' expected profits and the effects of cost uncertainty on market structure, it is necessary to transform the parameters  $a$ ,  $b$  and  $d$  back into terms of the parameters  $\alpha$ ,  $\beta$  and  $\gamma$  of the utility function (1). Making use of the latter, the relevant expected equilibrium profit is determined by:

$$E(\pi^*) = \frac{(\alpha - c)^2 (\beta - \gamma) [\beta + (n-2)\gamma]}{[\beta + (n-1)\gamma] [2\beta + (n-3)\gamma]^2} + \frac{[\beta + (n-2)\gamma] t (t+4u)}{4(\beta - \gamma) [\beta + (n-1)\gamma] (t+u)} \quad (17)$$

Both the derivatives of  $E(\pi^*)$  with respect to the cost deviation variance  $t$  and with respect to the signal variance  $u$  are positive. This means that expected equilibrium profits are positive functions of the degree of uncertainty as to unit cost and signal accuracy. Thus, firms can gain from higher cost uncertainty. Defining the combined cost uncertainty parameter  $T := \frac{t(t+4u)}{4(t+u)}$  which depends positively on both  $t$  and  $u$ , we obtain:

$$\frac{\partial E(\pi^*)}{\partial T} = \frac{\beta + (n-2)\gamma}{(\beta - \gamma) [\beta + (n-1)\gamma]^2} > 0 \quad (18)$$

In order to address the issue of market structure, we assume free entry into the market and introduce a fixed cost of entry. Consequently, the expected equi-

librium profits of all firms which enter the market are zero (not taking the integer problem into account). If the combined uncertainty parameter  $T$  is small enough to ensure that the regularity constraint of a non-negative markup of prices on the perceived unit cost is fulfilled (a sufficient regularity condition is given by  $T < \frac{4(\alpha-c)^2(\beta-\gamma)^2\{2\beta^2+4(n-2)\beta\gamma+[7+n(2n-7)]\gamma^2\}}{[2\beta+(n-3)\gamma]^3\gamma}$ ), then

$$\frac{\partial E(\pi^*)}{\partial n} < 0 \tag{19}$$

and the comparative statics yield a positive relationship between cost uncertainty and the number of entering firms. Since the expected equilibrium profit positively depends on market size  $\alpha$  and negatively on the fixed cost of entry, it can further be shown that the equilibrium number of firms in the market decreases with the level of entry cost but increases with the size of the market as well as with the degree of cost uncertainty.

## 4 Conclusion

This paper shows that concealing private cost information is a dominant strategy in a heterogeneous market with oligopolistic price competition. This unambiguous conclusion enables us to endogenize the number of firms in terms of market size, entry costs and unit cost uncertainty. If the degree of cost uncertainty is not too large, there exists a negative relationship between the entry cost and the number of firms, but a positive one between market size and the number of firms as well as between cost uncertainty and the number of firms. Finally, we observe a positive relationship between cost uncertainty and the endogenously determined number of firms.



## Appendix

In order to calculate the ex ante expected equilibrium profits (13), we make use of the means and variances of the deviation parameters  $\tau_i$ , and of the variances and means of the prices  $p_i$  and of the covariances  $\text{Cov}(\tau_i, p_i)$ :

$$\begin{aligned} E(p_i) &= \frac{a + bc}{2b - (n - 1)d} \\ \text{Var}(p_i) &= t^2 \left\{ \frac{1}{4(t + u)} + \frac{b^2 d^2}{(2b + d)^2 [2b - (n - 1)d]^2} \sum_{j \neq i} \frac{1}{t + u + r_j} \right. \\ &\quad \left. + \frac{(n - 1)d^2 [8b^2 - 4(n - 2)bd - (n - 1)d^2]}{4(2b + d)^2 [2b - (n - 1)d]^2} \frac{1}{t + u + r_i} \right\} \\ \text{Cov}(\tau_i; p_i) &= t^2 \left\{ \frac{1}{2(t + u)} + \frac{(n - 1)d^2}{2(2b + d)[2b - (n - 1)d]} \frac{1}{t + u + r_i} \right\} \end{aligned}$$

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