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by

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Abstract

This paper looks at the integration of regions and nations through the prism of the merger of populations (societies). The paper employs a particular index of social stress. Stylized examples of the merging of two populations suggest that with integration, the social stress index will increase. The examples form the basis for the development of new formulas for calculating the social stress of an integrated population as a function of the levels of social stress of the constituent populations when apart. The formulas reveal that the social stress of an integrated population is higher than the sum of the levels of social stress of the constituent populations when apart. This raises the distinct possibility that the merging of populations may be a social liability: integration may fail to give the populace a sense of improved wellbeing.

Keywords: Integration of regions and nations; Merging populations; Social welfare

JEL classification: D02; D63; F55; P51

1. Introduction

In this paper we look at the integration of regions and nations through a somewhat unusual prism. In what follows we do not strive to provide a balance sheet of the advantages and disadvantages of integration which, undoubtedly, include various efficiency and productivity gains. Rather, we seek to highlight a particular worrisome aspect of integration.

Integration and mergers of populations occur in various spheres of life. They may arise naturally or as a result of administrative considerations, they may be imposed or chosen. Conquests bring hitherto disparate populations into one, provinces consolidate into regions, small municipalities merge into a larger municipality (as is currently happening increasingly in Italy), adjacent villages that experience population growth coalesce into one town, schools and school classes are joined, firms concentrate production from two plants in one, branches of a bank amalgamate, East Germany and West Germany become united Germany, European countries integrate financially (adopting a common currency) and otherwise.

In general, when two populations merge, a variety of benefits are anticipated: denser markets, increased efficiency and productivity brought about by scale effects, and the like. Classical trade theory maintains that integration liberalizes trade and smoothes labor and financial flows. Larger markets improve resource allocation and the distribution of final products. The welfare of the integrating populations is bound to rise. Rivera-Batiz and Romer (1991) emphasize the influence of integration on the prevailing stock of knowledge and on the speed of technological advances, and van Elkan (1996) points to the role of integration in narrowing the technological gap between countries, which stimulates growth. Henrekson et al. (1997), who address the long-run growth effect of European integration, point to a particularly beneficial effect of integration.

The picture may not be so bright, however. Convergence in the income levels of the integrating countries or regions is not by any means inevitable. Behrens et al. (2007) show that to secure gains from integration, a significant degree of coordination of policies between countries is required, while Rivera-Batiz and Xie (1993), and Zeng and Zhao (2010) caution that the income inequality repercussions of integration may well depend on the characteristics of the countries or regions involved, which, when unfavorable, can result in increased inequality in the integrated population. Beckfield (2009), who studies European integration and individual levels of income, reports reduced between-country income inequality but increased within-country income inequality. The inconclusiveness of these outcomes also pervades research on firms: whereas Qiu and Zhou (2006) report increased profitability following the international merger of firms, Greenaway et al. (2008) point to a greater likelihood of a closedown when a firm faces tighter competition in a liberalized market. An interesting strand of literature deals with the merger of firms and workplaces, employing "social identity theory" (originally developed by Tajfel and Turner, 1979). A recurrent finding (cf. Terry et al., 2001; Terry and O'Brien, 2001; Fischer et al., 2007) is that different groups of individuals have contrasting perceptions: a merger is viewed most negatively by those of low status, whereas high status people are more at ease with the merged structure. This finding connects with one of the main claims of the current paper: when such contrasting perceptions are aggregated, belonging to a larger society results in a heightened level of social stress.

In this paper we employ a particular index of social stress, namely total relative deprivation, *TRD*, to assess the repercussions of a merger. In Sections 2, 3, and 4 we present the background, rationale, and logic for this index. In Section 5 we review stylized representations of mergers. We show that in each of two non-trivial scenarios, the index registers an increase. In Section 6 and 7 we develop new procedures for calculating the *TRD* of a merged population as a function of the *TRD*s of the constituent populations when apart. Building on these procedures we show that in a rich variety of settings, the *TRD* of a merged population is greater than the sum of the *TRD*s of the constituent populations when apart. Taking these steps raises the disturbing possibility, alluded to in Section 8, that, in and by itself, integration (for example, European monetary integration) may fail to reward the populace with a sense of improved wellbeing. In Section 9 we briefly conclude.

2. A measure of social stress

Consider a population N of n individuals whose incomes are $y_1 \leq y_2 \leq \ldots \leq y_n$, where $n \geq 2$. We measure the stress of an individual by relative deprivation, *RD*, which for an individual i who earns income y_i, where $i = 1,...,n-1$, and who refers to population N as his comparison group, is defined as

$$RD_N(y_i) \equiv \frac{1}{n}\sum_{k=i+1}^{n}(y_k - y_i), \tag{1}$$

and it is understood that $RD_N(y_n) = 0$.

The total relative deprivation of population N, TRD_N, is naturally the sum of the levels of relative deprivation of the individuals who belong to this population,

$$TRD_N = \sum_{i=1}^{n} RD_N(y_i) = \frac{1}{n}\sum_{i=1}^{n-1}\sum_{k=i+1}^{n}(y_k - y_i). \tag{2}$$

We resort to *TRD* as a measure of social stress of a population. In the next two sections we provide a brief account of the manner in which relative deprivation gained a foothold in economic analysis, and we explain in some detail how the measure of relative deprivation given in (1) is constructed.

3. A brief history of relative deprivation in economics

Considerable economic analysis has been inspired by the sociological-psychological concepts of *RD* and reference groups. Economists have come to consider these concepts as fitting tools for studying comparisons that affect an individual's behavior, in particular, comparisons with related individuals whose incomes are higher than his own income (cf. the large literature spanning from Duesenberry, 1949, to, for example, Clark et al., 2008). An individual has an unpleasant sense of being relatively deprived when he lacks a desired good and perceives that others in his reference group possess that good (Runciman, 1966).[1] Given the income distribution of the individual's reference group, the individual's *RD* is the sum of the deprivation caused by every income unit that he lacks (Yitzhaki, 1979; Hey and Lambert, 1980; Ebert and Moyes, 2000; Bossert and D'Ambrosio, 2006; Stark and Hyll, 2011).

The pioneering study in modern times that opened the flood-gate to research on *RD* and primary (reference) groups is the 1949 two-volume set of Stouffer et al. *Studies in*

[1] In Runciman's (1966) theory of *RD*, an individual's reference group is the group of individuals with whom the individual compares himself (cf. Singer, 1981).

Social Psychology in World War II: The American Soldier. That work documented the distress caused not by a given low military rank and weak prospects of promotion (military police) but rather by the pace of promotion of others (air force). It also documented the lesser dissatisfaction of black soldiers stationed in the South who compared themselves with black civilians in the South than the dissatisfaction of their counterparts stationed in the North who compared themselves with black civilians in the North. Stouffer's research was followed by a large social-psychological literature. Economics has caught up relatively late, and only somewhat. This is rather surprising because eminent economists in the past understood well that people compare themselves to others around them, and that social comparisons are of paramount importance for individuals' happiness, motivation, and actions. Even Adam Smith (1776) pointed to the social aspects of the necessities of life, and stressed the *relative* nature of poverty: "A linen shirt, for example, is, strictly speaking, not a necessary of life. The Greeks and Romans lived, I suppose, very comfortably, though they had no linen. But in the present times, through the greater part of Europe, a creditable day-laborer would be ashamed to appear in public without a linen shirt, the want of which would be supposed to denote that disgraceful degree of poverty […]" (p. 465). Marx's (1849) observations that "Our wants and pleasures have their origin in the society; [… and] they are of a relative nature" (p. 33) emphasize the social nature of utility, and the impact of an individual's relative position on his satisfaction. Inter alia, Marx wrote: "A house may be large or small; as long as the surrounding houses are equally small, it satisfies all social demands for a dwelling. But if a palace arises beside the little house, the house shrinks into a hut" (p. 33). Samuelson (1973), one of the founders of modern neoclassical economics, pointed out that an individual's utility does not depend only on what he consumes in *absolute* terms: "Because man is a social animal, what he regards as 'necessary comforts of life' depends on what he sees others consuming" (p. 218).

The relative income hypothesis, formulated by Duesenberry (1949), posits an asymmetry in the comparisons of income which affect the individual's behavior: the individual looks upward when making comparisons. Veblen's (1899) concept of *pecuniary emulation* explains why the behavior of an individual can be influenced by comparisons with the incomes of those who are richer. Because income determines the level of consumption, higher income levels may be the focus for emulation. Thus, an individual's income aspirations (to obtain the income levels of other individuals whose incomes are higher than his own) are shaped by the perceived consumption standards of the richer. In that way, invidious comparisons affect behavior, that is, behavior which leads to "the achievement of a favourable comparison with other men [...]" (Veblen, 1899, p. 33).[2]

4. The rationale and construction of a measure of social stress

Several recent insightful studies in social psychology (for example, Callan et al., 2011; Smith et al., 2012) document how sensing *RD* impacts negatively on personal wellbeing, but these studies do not provide a calibrating procedure; a sign is not a magnitude. For the purpose of constructing a measure, a natural starting point is the work of Runciman (1966), who, as already noted in the preceding section, argued that an individual has an unpleasant sense of being relatively deprived when he lacks a desired good and perceives that others with whom he naturally compares himself possess that good. Runciman (1966, p. 19) writes as follows: "The more people a man sees promoted when he is not promoted himself, the more people he may compare himself with in a situation where the comparison will make him feel deprived," thus implying that the deprivation from not having, say, income y is an increasing function of the fraction of people in the individual's reference group who have y. To aid intuition and for the sake of concreteness, we resort to income-based comparisons, namely an individual feels relatively deprived when others in his comparison group earn more than he does. An implicit assumption here is that the earnings of others are publicly known. Alternatively, we can think of consumption, which could be more publicly visible than income, although these two variables can reasonably be assumed to be strongly positively correlated.

[2] The empirical findings support the relative income hypothesis. Duesenberry (1949) found that individuals' savings rates depend on their positions in the income distribution, and that the incomes of the richer people affect the behavior of the poorer ones (but not *vice versa*). Schor (1998) showed that, keeping annual and permanent income constant, individuals whose incomes are lower than the incomes of others in their community save significantly less than those who are relatively better off in their community.

As an illustration of the relationship between the fraction of people possessing income y and the deprivation of an individual lacking y, consider a population (reference group) of six individuals with incomes $\{1,2,6,6,6,8\}$. Imagine a furniture store that in three distinct compartments sells chairs, armchairs, and sofas. An income of 2 allows you to buy a chair. To be able to buy any armchair, you need an income that is a little bit higher than 2. To buy any sofa, you need an income that is a little bit higher than 6. Thus, when you go to the store and your income is 2, what are you "deprived of?" The answer is "of armchairs," and "of sofas." Mathematically, this deprivation can be represented by $P(Y>2)(6-2)+P(Y>6)(8-6)$, where $P(Y>y_i)$ stands for the fraction of those in the population whose income is higher than y_i, for $y_i=2,6$. The reason for this representation is that when you have an income of 2, you cannot afford anything in the compartment that sells armchairs, and you cannot afford anything in the compartment that sells sofas. Because not all those who are to your right in the ascendingly ordered income distribution can afford to buy a sofa, yet they can all afford to buy armchairs, a breakdown into the two (weighted) terms $P(Y>2)(6-2)$ and $P(Y>6)(8-6)$ is needed. This way, we get to the very essence of the measure of *RD* used in much of this paper: we take into account the fraction of the comparison group (population) who possess some good which you do not, and we weigh this fraction by the "excess value" of that good. Because income enables an individual to afford the consumption of certain goods, we refer to comparisons based on income.

Formally, let $y=(y_1,...,y_m)$ be the vector of incomes in population N of size n with relative incidences $p(y)=\left(p(y_1),...,p(y_m)\right)$, where $m\leq n$ is the number of distinct income levels in y. The *RD* of an individual earning y_i is defined as the weighted sum of the excesses of incomes higher than y_i such that each excess is weighted by its relative incidence, namely

$$RD_N(y_i)\equiv\sum_{y_k>y_i}p(y_k)(y_k-y_i). \tag{3}$$

In the example given above with income distribution $\{1,2,6,6,6,8\}$, we have that the vector of incomes is $y=(1,2,6,8)$, and that the corresponding relative incidences are

$p(y)=(\frac{1}{6},\frac{1}{6},\frac{3}{6},\frac{1}{6})$. Therefore, the *RD* of the individual earning 2 is $\sum_{y_k>y_i} p(y_k)(y_k-y_i)=p(6)(6-2)+p(8)(8-2)$ $=\frac{3}{6}4+\frac{1}{6}6=3$. By similar calculations, we have that the *RD* of the individual earning 1 is higher at $3\frac{5}{6}$, and that the *RD* of each of the individuals earning 6 is lower at $\frac{1}{3}$.

We expand the vector y to include incomes with their possible respective repetitions, that is, we include each y_i as many times as its incidence dictates, and we assume that the incomes are ordered, that is, $y=(y_1,...,y_n)$ such that $y_1\le y_2\le ...\le y_n$. In this case, the relative incidence of each y_i, $p(y_i)$, is $1/n$, and, (3), defined for $i=1,...,n-1$, becomes just as given in (1)

$$RD_N(y_i)\equiv\frac{1}{n}\sum_{k=i+1}^{n}\left(y_k-y_i\right).$$

Looking at incomes in a large population, we can model the distribution of incomes as a random variable *Y* over the domain $[0,\infty)$ with a cumulative distribution function *F*. We can then express the *RD* of an individual earning y_i as

$$RD_N\left(y_i\right)=\left[1-F(y_i)\right]E\left(Y-y_i\mid Y>y_i\right). \tag{4}$$

To obtain this expression, starting from (3), we have that

$$\begin{aligned}
RD_N(y_i) &\equiv \sum_{y_k>y_i} p(y_k)(y_k-y_i)\\
&= \sum_{y_k>y_i} p(y_k)y_k - y_i\sum_{y_k>y_i} p(y_k)\\
&= [1-F(y_i)]\sum_{y_k>y_i}\frac{p(y_k)y_k}{1-F(y_i)} - y_i[1-F(y_i)]\\
&= [1-F(y_i)]E(Y\mid Y>y_i)-[1-F(y_i)]y_i\\
&= [1-F(y_i)]E(Y-y_i\mid Y>y_i).
\end{aligned}$$

The formula in (4) states that the *RD* of an individual whose income is y_i is equal to the product of two terms: $1-F\left(y_i\right)$, which is the fraction of those individuals in the population of n individuals whose incomes are higher than y_i, and $E(Y-y_i \mid Y>y_i)$, which is the mean excess income.

The formula in (4) is quite revealing because it casts *RD* in a richer light than the ordinal measure of rank or, for that matter, even the ordinal measure of status, which have been studied intensively in sociology and beyond. The formula informs us that when the income of individual A is, say, 10, and that of individual B is, say, 16, the *RD* of individual A is higher than when the income of individual B is 15, even though, in both cases, the rank of individual A in the income hierarchy is second. The formula also informs us that more *RD* is sensed by an individual whose income is 10 when the income of another is 14 (*RD* is 2) than when the income of each of four others is 11 (*RD* is $\frac{4}{5}$), even though the excess income in both cases is 4. This property aligns nicely with intuition: it is more painful (more stress is experienced) when the income of half of the population in question is 40 percent higher, than when the income of $\frac{4}{5}$ of the population is 10 percent higher. In addition, the formula in (4) reveals that even though *RD* is sensed by looking to the right of the income distribution, it is impacted by events taking place on the left of the income distribution. For example, an exit from the population of a low-income individual increases the *RD* of higher-income individuals (other than the richest) because the weight that the latter attach to the difference between the incomes of individuals "richer" than themselves and their own income rises. The often cited example from a three tenors concert organized for Wembley Stadium in which Pavarotti reputedly did not care how much he was paid so long as it was one pound more than Domingo was paid does not invalidate the logic behind our measure because, in light of the measure, Pavarotti's payment request can be interpreted as being aimed at ensuring that no *RD* will be experienced when he looks to the right in the pay distribution.

Similar reasoning can explain the demand for positional goods (Hirsch, 1976). The standard explanation is that this demand arises from the unique value of positional goods in elevating the social status of their owners ("These goods [are] sought after because they compare favorably with others in their class." Frank, 1985, p. 7). The distaste for relative deprivation offers another explanation: by acquiring a positional good, an individual shields himself from being leapfrogged by others which, if that were to happen, would expose him to *RD*. Seen this way, a positional good is a form of insurance against experiencing *RD*.

There can, of course, be other, quite intuitive ways of gauging *RD*, and in some contexts and for some applications, a measure simpler than (1) can be adequate. Suppose that an individual's income is *I*, and the average income of the individual's reference group is *R*. We can then define *RD* as a function of *I* and *R*, namely

$$RD(I,R) = \begin{cases} R - I & \text{if} \quad I < R \\ 0 & \text{if} \quad I \geq R. \end{cases} \tag{5}$$

This representation captures the intuitive requirements

$$\frac{\partial RD\left(I,R\right)}{\partial I} < 0, \ \frac{\partial RD\left(I,R\right)}{\partial R} > 0 \ \text{ for } R > I,$$

namely that, holding other things the same, for a relatively deprived individual (that is, for an individual whose income is lower than the average income of the individual's reference group), *RD* decreases with his own income, and increases with the average income of his reference group. Examples of the use of (5) are in Fan and Stark (2007), Stark and Fan (2011), and Stark and Jakubek (2013). However, the advantage of using (1) is that it is based on an axiomatic foundation which is, essentially, a translation of Runciman's (1966) work, let alone that it is nice in economics to draw on a foundation laid out in social psychology.

The formula in (4) that the *RD* of an individual is equal to the product of the fraction of those in the population whose incomes are higher than his and the mean excess income, was derived for income as a discrete variable. For the sake of completeness, we note that the formula applies just as well to income considered as a continuous variable. To see this, let

$$F(y) = \int_{-\infty}^{y} f(t)\,dt.$$

Because

$$f(x|x>y)=\frac{f(x)}{1-F(y)},$$

it follows that

$$\int_y^\infty f(x)\,x\,dx=[1-F(y)]\int_y^\infty f(x|x>y)x dx=[1-F(y)]E(x|x>y).$$

Thus,

$$RD(y)\equiv\int_y^\infty f(x)(x-y)\,dx=\int_y^\infty f(x)\,x\,dx+\int_y^\infty f(x)(-y)\,dx$$

$$=[1-F(y)]\,E(x|x>y)-y\int_y^\infty f(x)dx.$$

Because

$$-y\int_y^\infty f(x)dx=-y[\int_{-\infty}^\infty f(x)dx-\int_{-\infty}^y f(x)dx]=-y[1-F(y)],$$

we get that

$$RD(y)=[1-F(y)]\,E(x|x>y)-y\int_y^\infty f(x)dx=[1-F(y)]\,E(x|x>y)-y[1-F(y)]$$

$$=[1-F(y)]\,E(x-y|x>y).$$

We now move from a theoretical background account to consider several specific income distributions and to assess how a merger impacts on the relative deprivation experienced by the integrated population.

5. Three scenarios for the mergers of populations

5.1 Scenario 1: a merger of two identical populations

For the sake of ease of reference, we will use a slightly different notation of the income distribution of a population. Let there be two populations, A and B, with income distributions $I_A\{1,2\}$, and $I_B\{1,2\}$. Then, $TRD_A\{1,2\} = TRD_B\{1,2\} = \frac{1}{2}$. Let the two populations merge. The income distribution of the merged population is $I_{A \cup B}\{1,1,2,2\}$. Summing over the post-merger *RD* of the individuals, we get that $TRD_{A \cup B} = \frac{2}{4} + \frac{2}{4} = 1$. Each of the two individuals whose income is 1 continues to experience exactly the same level of relative deprivation as prior to the merger, and the *TRD* of the merged population is twice what it was in each of the constituent populations when apart. Notably, the act of merging results in concentrating in one population the relative deprivation that was distributed between the constituent populations prior to the merger.

5.2 Scenario 2: merger of two different populations whose income distributions do not overlap

Let there be two populations, a relatively poor population *C* with income distribution $I_C\{1,2\}$, and a relatively rich population *D* with income distribution $I_D\{4,4\}$. Then, $TRD_C\{1,2\} = \frac{1}{2}$, and $TRD_D\{4,4\} = 0$. Let the two populations merge. The income distribution of the merged population is $I_{C \cup D}\{1,2,4,4\}$. Summing over the post-merger *RD* of the individuals, we get that $TRD_{C \cup D} = \frac{7}{4} + \frac{4}{4} = \frac{11}{4}$; the *TRD* of the merged population is five and a half times what it was in the constituent population *C*, and infinite times what it was in the constituent population *D*. The *TRD* of the merged population is higher than the sum of the *TRD*s of the constituent populations when apart.

5.3 Scenario 3: merger of two different populations whose income distributions overlap

Let there be two populations, population *G* with income distribution $I_G\{1,6\}$, and population *H* with income distribution $I_H\{4,5\}$, namely the income distribution of *H* is "immersed" en block in the income distribution of *G*. Then, $TRD_G\{1,6\} = \frac{5}{2}$, and $TRD_H\{4,5\} = \frac{1}{2}$. The merger of *G* and *H* yields income distribution $I_{G \cup H}\{1,4,5,6\}$.

Summing over the post-merger *RD* of the individuals, we get that $TRD_{G\cup H} = \frac{12}{4} + \frac{3}{4} + \frac{1}{4} = 4$; the *TRD* of the merged population is three fifths higher than what it was in the constituent population *G*, and eight times higher than it was in the constituent population *H*. The *TRD* of the merged population is higher than the sum of the *TRD*s of the constituent populations when apart.[3]

6. Calculating the *TRD* of a merged population as a function of the *TRD*s of the constituent populations

6.1 The *TRD* when non-overlapping populations merge

Let there be two populations, *M* and *N*, and let there be m individuals in population *M*, and n individuals in population *N*. We denote the total relative deprivation of each of these two populations when apart by TRD_M and TRD_N, respectively. Let the incomes of the individuals in *M* be $x_1 \le x_2 \le ... \le x_m$, and let the incomes of the individuals in *N* be $y_1 \le y_2 \le ... \le y_n$, such that the highest income in population *M* is lower than the lowest income in population *N*, namely $x_m < y_1$. Thus, population *M* is relatively poor, whereas population *N* is relatively rich. We denote the mean incomes of populations *M* and *N* by μ_M and μ_N, respectively. Obviously, $\mu_M < \mu_N$. Then, we have the following claim.

[3] A commentator on an earlier version of this paper stated: "I think it is intuitive that if we combine populations, the resulting merged population will be more heterogeneous than the first ones." As a matter of fact, the opposite holds. To see why intuition alone is not all that revealing, consider what is presumably the most intuitive measure of heterogeneity, namely the population variance: $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$, where $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$. The formula for σ^2 can be used even when the "randomness" of the observations is not well known, or even when we do not care much about the probabilistic nature of the observations - it simply can be used as a measure of spread (heterogeneity) among an arbitrarily given set of numbers. Then, we can think of the observations as a random variable which takes the values $x_1,...,x_n$ with uniform probabilities $\frac{1}{n}$. Given this, in the case of Scenario 3, the variance of income distribution $I_G\{1,6\}$ is 6.25; the variance of income distribution $I_H\{4,5\}$ is 0.25; and the variance of merged income distribution $I_{G\cup H}\{1,4,5,6\}$ is 3.5. That is, the variance of the merged population is smaller than the sum of the variances of the constituent populations when apart. This is exactly the opposite of the *TRD* result.

Claim 1: Denoting by $TRD_{M\cup N}$ the total relative deprivation of the merged population of *M* and *N*, $TRD_{M\cup N}=\frac{mTRD_M}{m+n}+\frac{(\mu_N-\mu_M)mn}{m+n}+\frac{nTRD_N}{m+n}$.

Proof: From the assumption that $x_m<y_1$, we know that the individuals from *N* do not feel relatively deprived of incomes of the individuals in *M*. Using this fact, and the definition of *TRD* in (2), we have that

$$TRD_{M\cup N}=\frac{1}{m+n}\left[\sum_{i=1}^{m-1}\sum_{j=i+1}^{m}(x_j-x_i)+\sum_{k=1}^{n-1}\sum_{l=k+1}^{n}(y_l-y_k)+\sum_{i=1}^{m}\sum_{k=1}^{n}(y_k-x_i)\right]. \tag{6}$$

The first two double sums in (6) are clearly $mTRD_M$ and $nTRD_N$, respectively, whereas the third double sum in (6) is that part of the *TRD* of the poorer population *M* which arises from the comparisons with the richer population *N*. We know that

$$\begin{aligned}\sum_{i=1}^{m}\sum_{k=1}^{n}(y_k-x_i)&=\sum_{i=1}^{m}(\sum_{k=1}^{n}y_k-nx_i)=n\sum_{i=1}^{m}(\mu_N-x_i)\\&=n(m\mu_N-\sum_{i=1}^{m}x_i)=mn(\mu_N-\mu_M)\,.\end{aligned} \tag{7}$$

Thus, from inserting (7) into (6), we get that

$$TRD_{M\cup N}=\frac{1}{m+n}\left[mTRD_M+nTRD_N\right]+\frac{mn(\mu_N-\mu_M)}{m+n}.$$

From inspection of the expression of $TRD_{M\cup N}$ in Claim 1, we get that $TRD_{M\cup N}$ is higher the larger is $(\mu_N-\mu_M)$, and, for a given aggregate size $m+n$, the larger is m (*n*) if $TRD_M>TRD_N$ ($TRD_N>TRD_M$). These results are intuitively appealing: the farther apart the constituent populations on average, the larger the increase in *TRD* upon a merger; and the larger the relative size of the constituent population with the higher *TRD*, the larger the increase in the *TRD* of the merged population.

6.2 The *TRD* when populations of any type (overlapping or non-overlapping) merge

We next relax the assumption that the two populations do not necessarily overlap. As before, we have population *M* of *m* individuals, and population *N* of *n* individuals, and the income distributions in the two populations are given, respectively, by $x_1 \leq x_2 \leq ... \leq x_m$ and $y_1 \leq y_2 \leq ... \leq y_n$. However, we now allow for the possibility that the highest income in population *M*, x_m, is higher than the lowest income in population *N*, y_1. We then have the following claim.

Claim 2: Denoting by $TRD_{M \cup N}$ the total relative deprivation of the merged population of *M* and *N*, $TRD_{M \cup N} = \frac{mTRD_M}{m+n} + \frac{\sum_{i=1}^{m}\sum_{k=1}^{n} | x_i - y_k |}{m+n} + \frac{nTRD_N}{m+n}$.

Proof: In order to prove the claim, we first rewrite TRD in a form that allows for a nicer mathematical treatment.

Lemma 1: Let a population M of m individuals with incomes $x_1 \leq ... \leq x_m$ be given. Then,

$$TRD_M = \frac{1}{2m} \sum_{k=1}^{m} \sum_{i=1}^{m} \left| x_k - x_i \right|. \tag{8}$$

Proof of Lemma 1: For all $i, k = 1, \quad , m$ either $x_k - x_i \geq 0$, or $x_i - x_k \geq 0$. TRD in (2) includes only non-negative differences between incomes in a distribution. Because the *TRD* in (8) includes the absolute values of *all* the differences between incomes, it counts a difference between a pair of given incomes twice.

Thus, we have that

$$\frac{1}{m} \sum_{i=1}^{m} \sum_{k=1}^{m} \left| x_k - x_i \right| = 2 \frac{1}{m} \sum_{i=1}^{m-1} \sum_{k=i+1}^{m} (x_k - x_i). \tag{9}$$

Inserting (9) into (2), we obtain (8). □

We now use Lemma 1 to prove the claim. We consider how TRD "behaves" upon the merging of two populations that may overlap. Using (8), we have that

$$TRD_{M\cup N} = \frac{1}{2(n+m)}\left[\sum_{i=1}^{m}\sum_{j=1}^{m}\left|x_j - x_i\right| + \sum_{k=1}^{n}\sum_{l=1}^{n}\left|y_l - y_k\right| + 2\sum_{i=1}^{m}\sum_{k=1}^{n}\left|x_i - y_k\right|\right]. \quad (10)$$

The first two double sums in (10) are clearly $2mTRD_M$ and $2nTRD_N$, respectively. We therefore have that

$$TRD_{M\cup N} = \frac{1}{m+n}\left[mTRD_M + nTRD_N\right] + \frac{1}{m+n}\sum_{i=1}^{m}\sum_{k=1}^{n}\left|x_i - y_k\right|.$$

7. Comparing the *TRD* of a merged population with the sum of the *TRD*s of the constituent populations when apart

7.1 The relationship between the *TRD* of a merged population and the sum of the *TRD*s of any two constituent populations of two individuals each

We next seek to show that the merger of two populations each consisting of two individuals results in the *TRD* of the merged population being higher than the sum of the *TRD*s of the constituent populations. This is not an intuitively obvious result even in the simple case in which the two populations do not overlap and a relatively poor two-individual population merges with a relatively rich two-individual population. In such a case, it is quite clear that upon integration the individuals from the poorer population are subjected to more relative deprivation, whereas (assuming that the incomes of the two rich individuals differ) the individuals from the richer population, except the richest, are subjected to less relative deprivation. Because one constituent population experiences an increase in *TRD* while the other constituent population experiences a decrease, whether the *TRD* of the merged population is higher than the sum of the *TRD*s of the constituent populations cannot be ascertained without additional formal analysis. To this end, we now state and prove the following claim.

Claim 3: Let there be two populations of two individuals each: population *A*, and population *B*. Let the incomes of the four individuals be distinct. A merger of the two populations results in an increase of *TRD*, that is, $TRD_{A\cup B} > TRD_A + TRD_B$.

Proof: The proof is in the Appendix.

Corollary: In the degenerate case in which the incomes of population *A* are identical to the incomes of population *B*, $TRD_{A\cup B} = TRD_A + TRD_B$.

Proof: When the incomes of population *A* are identical to the incomes of population *B*, merging the two populations is equivalent to doubling the number of income recipients of each income. Without loss of generality, let each of the two populations consist of two individuals with incomes 1 and $1+\alpha$, where $\alpha > 0$. Because *TRD* is a measure with homogeneity of degree one, it follows that

$TRD_{A\cup B} = TRD\{1,1,1+\alpha,1+\alpha\} = TRD\{2\times 1, 2\times(1+\alpha)\} = 2TRD\{1,1+\alpha\} = TRD_A + TRD_B$.

Scenario 1 considered in Section 5 constitutes such a case.

7.2 The relationship between the *TRD* of a merged population and the sum of the *TRD*s of two possibly overlapping constituent populations of the same number of individuals, *n*, for any $n \geq 2$

Scenario 3 demonstrated that the merger of a relatively poor (in terms of income per capita) two-individual population with a relatively rich (in terms of income per capita) two-individual population when the two populations overlap results in a *TRD* of the merged population that is higher than the sum of the *TRD*s of the constituent populations. Drawing on Claim 2, we next show that the merging of equally-sized overlapping populations (or, for that matter, non-overlapping populations) results in a *TRD* of the merged population that is higher than the sum of the *TRD*s of the constituent populations - a generalization of Claim 3. To this end, we first state and prove the following lemma.

Lemma 2: Let $u \leq v$ and $r \leq s$ be real numbers. Then,

$$|u-s| + |v-r| \geq (v-u) + (s-r). \tag{11}$$

Proof of Lemma 2: Given that $u \leq v$ and $r \leq s$, there are six possible orderings of these numbers. We consider each case separately.

1. $u \le v \le r \le s$. Then,

   $|u-s|+|v-r| \ge (s-u) = (s-r)+(r-v)+(v-u) \ge (v-u)+(s-r)$. The case where $r \le s \le u \le v$ follows by symmetry.
2. $u \le r \le v \le s$. Then, $|u-s|+|v-r| = (s-u)+(v-r) = (v-u)+(s-r)$. The case where $r \le u \le s \le v$ follows by symmetry.
3. $u \le r \le s \le v$. Then, $|u-s|+|v-r| = (s-u)+(v-r) = (v-u)+(s-r)$. The case where $r \le u \le v \le s$ follows by symmetry.

We now use Lemma 2 to prove the following claim.

Claim 4: Let there be two populations, *M* and *N*, with the same number of individuals in each population. Then, $TRD_{M \cup N} \ge TRD_M + TRD_N$.

Proof: Let there be *n* individuals in each population, let the incomes of the individuals in *M* be $x_1 \le \ldots \le x_n$, and let the incomes of the individuals in *N* be $y_1 \le \ldots \le y_n$. From (2) we know that

$$TRD_M = \frac{1}{n}\sum_{i=1}^{n-1}\sum_{k=i+1}^{n}(x_k - x_i)$$

and, similarly, that

$$TRD_N = \frac{1}{n}\sum_{i=1}^{n-1}\sum_{k=i+1}^{n}(y_k - y_i).$$

From Claim 2 we also get that

$$TRD_{M \cup N} = \frac{1}{2}(TRD_M + TRD_N) + \frac{1}{2n}\sum_{i=1}^{n}\sum_{k=1}^{n}|x_i - y_k|. \tag{12}$$

Drawing on Lemma 2 with $x_i \le x_k$ and $y_i \le y_k$, and leaving out the terms with i=k, we have that

$$\frac{1}{2n}\sum_{i=1}^{n}\sum_{k=1}^{n}|x_i - y_k| \ge \frac{1}{2n}\sum_{i=1}^{n-1}\sum_{k=i+1}^{n}(|x_i - y_k| + |x_k - y_i|) \ge \frac{1}{2n}\sum_{i=1}^{n-1}\sum_{k=i+1}^{n}[(x_k - x_i) + (y_k - y_i)]. \tag{13}$$

The most right hand side term in (13) is equal to $\frac{1}{2}(TRD_M + TRD_N)$. Thus, we have that

$$\frac{1}{2n}\sum_{i=1}^{n}\sum_{k=1}^{n}|x_i - y_k| \geq \frac{1}{2}(TRD_M + TRD_N). \tag{14}$$

Upon inserting (14) into (12), we get that

$$TRD_{M\cup N} \geq TRD_M + TRD_N.$$

7.3 The relationship between the *TRD* of a merged population and the sum of the *TRD*s of two non-overlapping constituent populations of different numbers of individuals

Claim 5: Let there be two populations, M of m individuals with incomes $x_1 \leq x_2 \leq \quad \leq x_m$, and N of n individuals with incomes $x_{m+1} \leq x_{m+2} \leq \quad \leq x_{m+n}$, such that $x_m < x_{m+1}$. Then, $TRD_{M\cup N} > TRD_M + TRD_N$.

Proof: Upon a merger of M and N, there will be a population $M \cup N$ with incomes $\{x_1, x_2, \quad, x_m, x_{m+1}, x_{m+2}, \quad, x_{m+n}\}$. We seek to show that

$$\begin{aligned} TRD_{M\cup N} &= \frac{mn\sum_{i=1}^{m-1}\sum_{j=i+1}^{m}(x_j - x_i)}{(m+n)mn} + \frac{m^2n^2(\frac{\sum_{j=m+1}^{m+n}x_j}{n} - \frac{\sum_{i=1}^{m}x_i}{m})}{(m+n)mn} + \frac{mn\sum_{i=m+1}^{m+n-1}\sum_{j=i+1}^{m+n}(x_j - x_i)}{(m+n)mn} \\ &> \frac{n(m+n)\sum_{i=1}^{m-1}\sum_{j=i+1}^{m}(x_j - x_i)}{(m+n)mn} + \frac{m(m+n)\sum_{i=m+1}^{m+n-1}\sum_{j=i+1}^{m+n}(x_j - x_i)}{(m+n)mn} = TRD_M + TRD_N, \end{aligned} \tag{15}$$

where the three terms in the first row of (15) are $TRD_{M\cup N}$ decomposed in line with Claim 1, and the two terms in the second row of (15) are TRD_M and TRD_N, respectively, with all five terms reduced to a common denominator. From (7), we know that

$$mn(\frac{\sum_{j=m+1}^{m+n}x_j}{n} - \frac{\sum_{i=1}^{m}x_i}{m}) = \sum_{i=1}^{m}\sum_{j=m+1}^{m+n}(x_j - x_i) \ .$$

Thus, seeking to show that (15) holds is equivalent to seeking to show that

$$\Phi = mn\sum_{i=1}^{m}\sum_{j=m+1}^{m+n}(x_j - x_i) - n^2\sum_{i=1}^{m-1}\sum_{j=i+1}^{m}(x_j - x_i) - m^2\sum_{i=m+1}^{m+n-1}\sum_{j=i+1}^{m+n}(x_j - x_i) > 0\,. \tag{16}$$

It is easy to see that

$$\begin{aligned}\sum_{i=1}^{m-1}\sum_{j=i+1}^{m}(x_j - x_i) = (x_2 - x_1) + (x_3 - x_1) + \quad &+ (x_m - x_1)\\ + (x_3 - x_2) + \quad &+ (x_m - x_2)\\ \\ &+ (x_m - x_{m-1}),\end{aligned}$$

or that

$$\begin{aligned}\sum_{i=1}^{m-1}\sum_{j=i+1}^{m}(x_j - x_i) &= (m-1)x_m + (m-2)x_{m-1} + \quad + 2x_3 + x_2\\ &\quad - (m-1)x_1 - (m-2)x_2 - \quad - 2x_{m-2} - x_{m-1}\\ &= (m-1)x_m + (m-3)x_{m-1} + \quad + (3-m)x_2 + (1-m)x_1,\end{aligned} \tag{17}$$

and that

$$\begin{aligned}\sum_{i=1}^{m}\sum_{j=m+1}^{m+n}(x_j - x_i) &= (x_{m+1} - x_1) + (x_{m+2} - x_1) + \quad + (x_{m+n} - x_1)\\ &\quad + (x_{m+1} - x_2) + (x_{m+2} - x_2) + \quad + (x_{m+n} - x_2)\\ \\ &\quad + (x_{m+1} - x_m) + (x_{m+2} - x_m) + \quad + (x_{m+n} - x_m)\\ &= m(x_{m+1} + x_{m+2} + \quad + x_{m+n}) - n(x_1 + x_2 + \quad + x_m).\end{aligned} \tag{18}$$

Using (17) for the second and third terms in (16), and (18) for the first term in (16), we can then rewrite (16) as

$$\begin{aligned}\Phi = {}& m^2n(x_{m+1} + x_{m+2} + \quad + x_{m+n}) - mn^2(x_1 + x_2 + \quad + x_m)\\ & - n^2[(m-1)x_m + (m-3)x_{m-1} + \quad + (3-m)x_2 + (1-m)x_1]\\ & - m^2[(n-1)x_{m+n} + (n-3)x_{m+n-1} + \quad + (3-n)x_{m+2} + (1-n)x_{m+1}] > 0,\end{aligned}$$

or, after rearranging, as

$$
\begin{aligned}
\Phi &= m^2[x_{m+n} + 3x_{m+n-1} + \quad + (2n-3)x_{m+2} + (2n-1)x_{m+1}] \\
&\qquad - n^2[(2m-1)x_m + (2m-3)x_{m-1} + \quad + 3x_2 + x_1] \\
&\geq m^2[x_{m+1} + 3x_{m+1} + \quad + (2n-3)x_{m+1} + (2n-1)x_{m+1}] \\
&\qquad - n^2[(2m-1)x_m + (2m-3)x_m + \quad + 3x_m + x_m] \\
&= m^2n^2x_{m+1} - m^2n^2x_m > 0.
\end{aligned}
$$

8. Social welfare implications

Making comparisons of social welfare is notoriously difficult. Still, with the aid of simple auxiliary assumptions, we are able to assess the welfare repercussions of the merger of populations. In general, to render a welfare judgement, it is necessary to identify what to compare, and how to measure it. In what follows, we compare the social welfare, *SW*, of each constituent population following integration with the social welfare experienced by the population prior to integration. We assume that the *SW* of a population is a function of per capita income and of per capita *TRD* (per capita social stress), with the partial first derivates being, respectively, positive and negative. Because throughout we have kept incomes unchanged, the incomes of the individuals of a constituent population are not affected by its merger with another population and, hence, (as there is neither population growth nor population decline), the per capita income of every individual of a constituent population remains unchanged. This point is worth reiterating: in our setting, a merger changes the space of social comparisons that governs the sensing and calculation of relative income (relative deprivation), but it leaves absolute incomes intact. Thus, the integration-caused change in the *SW* of a constituent population is related only to the change in per capita *TRD*. In the following comparisons of changes in the *SW* upon integration, we consider *TRD* in total (not in per capita terms) because the number of individuals in each constituent population remains unchanged.

We first study the *SW* repercussions of each of the three scenarios presented in Section 5. Thereafter, we provide several generalizations.

8.1 The three scenarios, and a little beyond

In Scenario 1, the *TRD* of population *A* after the merger is the same as it was prior to the merger, namely $TRD_{A|A\in A\cup B} = TRD_A = \frac{1}{2}$. And likewise with regard to population *B*. We conclude that social welfare remains intact.

In Scenario 2, integration with population *D* with income distribution $I_D\{4,4\}$ exposes individuals of population *C* with income distribution $I_C\{1,2\}$ to higher *TRD*: $TRD_{C|C\in C\cup D} = \frac{11}{4} > TRD_C = \frac{1}{2}$. Thus, population *C* experiences a decline in *SW*. The *SW* of population *D* remains unchanged at 0 upon the merger. We conclude that integration entails a reduction in social welfare.

In Scenario 3, integration exposes each of the two constituent populations (with income distributions $I_G\{1,6\}$, and $I_H\{4,5\}$) to higher *TRD*: $TRD_{G|G\in G\cup H} = \frac{3}{4}+\frac{4}{4}+\frac{5}{4} = 3 > TRD_G = \frac{5}{2}$; $TRD_{H|H\in G\cup H} = \frac{3}{4}+\frac{1}{4} = 1 > TRD_H = \frac{1}{2}$. We conclude that in this case too, integration entails a reduction in social welfare.

Taking a clue from these three scenarios, we can generalize intuitively as follows: when there are two populations, say *M* and *N*, of two individuals each, it can never be the case that there is a universal welfare gain, namely that the *TRD*s of *both* populations are lowered upon a merger: either the *TRD*s of both populations remain unchanged (as in Scenario 1), or the *TRD* of at least one population increases (as in Scenarios 2 and 3).

Ruling out the case in which all the incomes in *N* are identical, when populations *M* and *N do not overlap* and, without loss of generality, *M* is relatively poorer, a merger must reduce the *TRD* of *N*. The converse applies to population *M*.

When populations *M* and *N overlap*, then, without loss of generality, either *M* mingles with *N*, or it is "immersed" en bloc in *N* (as in Scenario 3). Regarding the first case, that is, if $I_M\{x_1,x_2\}$, $I_N\{y_1,y_2\}$, and the sequence is $x_1 < y_1 < x_2 < y_2$, then, as is easy to ascertain, the *TRD* of population *M* upon a merger, namely $TRD_{M|M\in M\cup N}$, must be higher than the *TRD* of population *M* prior to the merger, and the *TRD* of population *N* upon a merger, namely $TRD_{N|N\in M\cup N}$, must be lower than the *TRD* of population *N* prior to the merger.[4] To wit, given the sequence as above, $x_1 < y_1 < x_2 < y_2$, it can never be the case that both populations record a decrease in their *TRD*s.

An analogous analysis of changes in *TRD*s experienced by two populations in the wake of integration when one is "immersed" en bloc in the other,[5] leads to the claim that a merger of two populations of two individuals each can never confer a universal social welfare gain upon both populations.

The preceding discussion leads us to the following generalizations.

8.2 A change in social welfare following merger when each of the merged populations consists of two or more individuals

8.2.1 Non-overlapping populations

When the merger is of *any* two populations *M* and *N* that *do not overlap* (by "*any*" we mean that the size of *M* is $m \geq 2$, and that the size of *N* is $n \geq 2$) such that, without loss of generality, *M* is relatively poorer (and ruling out the case in which all the incomes in *N* are identical), a merger must reduce the *TRD* of *N*, namely it lowers *N*'s social stress; consequently, this population experiences a social welfare gain. The converse applies to population *M*. In the general non-overlapping case then, and unlike in the three scenarios considered, we might not be able to end up with an unequivocal global welfare judgment because one population gains while the other loses, and it is not up to us to assign weights to these contrasting changes. However, if we make a global welfare judgment on the

[4] Formally, assume that the incomes of population *M* are 1 and $1+a+b$, while the incomes of population *N* are $1+a$ and $1+a+b+c$, where *a*, *b* and *c* are arbitrary positive constants. Then, $TRD_{M|M\in M\cup N} = \frac{3a+2b+2c}{4} > \frac{2a+2b}{4} = \frac{a+b}{2} = TRD_M$, namely upon a merger there is an increase in the social stress of population *M*. At the same time, population *N* experiences a post-merger decrease in social stress: $TRD_{N|N\in M\cup N} = \frac{2b+c}{4} < \frac{2b+2c}{4} = \frac{b+c}{2} = TRD_N$.

[5] Denote the incomes of population *M* as $1+a$ and $1+a+b$, and the incomes of population *N* as 1 and $1+a+b+c$, where *a*, *b* and *c* are arbitrary positive constants. Then, $TRD_{M|M\in M\cup N} = \frac{2b+2c}{4} > \frac{2b}{4} = \frac{b}{2} = TRD_M$, namely upon a merger there is an increase in the social stress of population *M*. The social welfare repercussions of the merger for population *N* cannot be assessed easily, because they depend on the differences between the incomes of the individuals of both populations. Prior to the merger $TRD_N = \frac{a+b+c}{2} = \frac{2a+2b+2c}{4}$, whereas following the merger $TRD_{N|N\in M\cup N} = \frac{3a+2b+c}{4}$.

basis of a comparison of the *TRD* of the merged population with the sum of the *TRD*s of the constituent populations *and* assign equal weights in the sum of the *TRD*s of the two populations to each of the *TRD*s of the constituent populations, then we have the following claim.

Claim 6: The *SW* of two non-overlapping constituent populations under a merger is lower than the sum of the *SW*s of the constituent populations when apart.

Proof: Cf. the proof of Claim 5.

8.2.2 Overlapping populations

The study of the case in which populations *M* and *N* overlap is more difficult. Still, we can make some headway.

Claim 7: The following statement is false: "when the merger is of two overlapping populations, both populations experience a welfare gain".

Proof: The proof is by example, cf. Scenario 3.

Claim 8: When the merger is of two overlapping populations of the same size, it is never the case that both populations experience a welfare gain.

Proof: If both populations were to experience a welfare gain, then it would have to be the case that $TRD_{M \cup N} < TRD_M + TRD_N$. But from Claim 4 we know that the opposite holds, that is, that $TRD_{M \cup N} \geq TRD_M + TRD_N$.

If, akin to the case of non-overlapping populations discussed in Section 8.2.1 that led to Claim 6, we were to make a global welfare judgment on the basis of a comparison of the *TRD* of the merged population with the sum of the *TRD*s of the constituent populations *and* maintain a stand of cross-population impartiality (neutrality), that is, assign equal weights to each of the *TRD*s of the constituent populations in the sum of the *TRD*s of the two populations, then we will have the following claim.

Claim 9: The *SW* of the two constituent populations of the same number of individuals under a merger is lower than or equal to the sum of the *SW*s of the constituent populations when apart.

Proof: Cf. the proof of Claim 4.

9. Conclusion

As already noted in Section 1, mergers of populations occur in all spheres of life, and in all times and places. Mergers may arise as a result of administrative considerations or naturally, they may be imposed or chosen by election. A merger of populations is a far cry from the merger of production lines. The social environment and the social horizons that the individuals who constitute the merged population face change fundamentally upon a merger: others who were previously outside the individuals' social domain are now within. One consequence of this revision of the social landscape, which hitherto appears not to have received due attention, is a built-in increase in social stress: in a rich variety of settings, we have shown that the *TRD* of a merged population is larger than the sum of the *TRD*s of the constituent populations when apart. As a consequence, integration can fail to reward the populace with a sense of improved wellbeing and damage social harmony in quite unexpected ways.

Appendix

Proof of Claim 3: With all incomes distinct (pairwise different) we assume, without loss of generality, that the smallest income is 1 and that it is obtained in population *A*. Thus, the incomes in population *A* are

$$1,\ 1+\alpha,$$

and the incomes in population *B* are

$$1+\beta,\ 1+\beta+\delta,$$

where $\alpha, \beta, \delta > 0$ are arbitrary. Clearly,

$$TRD_A = \frac{\alpha}{2}, \text{ and } TRD_B = \frac{\delta}{2}.$$

To evaluate the *TRD* of the four-individual population *C* with incomes

$$1, 1+a, 1+a+b, 1+a+b+c$$

and with arbitrary $a, b, c > 0$, we note, referring to the four individuals as (1), (2), (3), and (4), that

$$RD(1) = \frac{1}{4}[a+(a+b)+(a+b+c)],\ RD(2) = \frac{1}{4}[b+(b+c)], RD(3) = \frac{c}{4}, RD(4) = 0.$$

Therefore,

$$TRD_C = RD(1) + RD(2) + RD(3) = \frac{1}{4}(3a+4b+3c). \tag{A1}$$

We now consider the *TRD* of $A \cup B$. Depending on the relative magnitudes of α, β, δ we have three cases: $\alpha < \beta$; $\beta < \alpha < \beta + \delta$; and $\alpha > \beta + \delta$. We attend to the second case; the proof of the other two cases is analogous.

When $\beta < \alpha < \beta + \delta$, we have that $\alpha = \beta + \varepsilon$ for some $\varepsilon > 0$. Consequently, we arrange the incomes as

$$1, 1+\beta, 1+\beta+\varepsilon, 1+\beta+\varepsilon+(\delta-\varepsilon),$$

and we note, because $\beta + \delta > \alpha$, that $\delta - \varepsilon > 0$. Using this and (A1),

$$TRD_{A\cup B} = \frac{1}{4}[3\beta + 4\varepsilon + 3(\delta - \varepsilon)] > \frac{1}{4}(3\beta + 2\varepsilon + 2\delta) > \frac{\beta + \varepsilon}{2} + \frac{\delta}{2} = \frac{\alpha}{2} + \frac{\delta}{2} = TRD_A + TRD_B.$$

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